1.

a) **M’ = {Q U {qnew}, Σ, Γ, δ’, q0, qnew}**

**where qnew is a new state**

**δ’((q, x, y)) = { δ((q,x,y)) if q ∈ Q\F, x ∈ Σε, y ∈ Γε**

**δ((q,x,y)) if q ∈ F, x ∈ Σε \ {a}, y ∈ Γ**

**{(qnew, ε)} U δ((q,x,y)) if q ∈ F, x = a, y = ε**

**∅ otherwise }**

**Our set of states is the states of M along with our new accept state**

**All our old accept states point to our new accept state**

**Everything else stays the same**

b)

**G’ = {V U {W}, Σ U a, R U {W -> Sa}, W}**

**Where W is a variable not in V.**

**We Σ U a in case a is not included in G, use the same rules as R, but our new rule and start state is the one that appends a to the end of languages defined by G.**

2.

a) **({S, T, V}, {a, b}, R, S)**

**with rules R = { S -> aT | bT | ε**

**T -> aV | bV**

**V -> aS | bS }**

b) **It is not ambiguous as we can’t find more than one leftmost derivation. Every leftmost step we take cannot be used ambiguously to create the same string.**

**e.g.**

**If we are trying to form the string aab**

**if S -> aT -> aaV -> aabS -> aab**

**Each step is the only step we can take for the sake of our string, as it must be used to form the next symbol required.**

c) **({S}, {a, b}, R, S)**

**with rules R = { S -> SaSaSbS | SaSbSaS | SbSaSaS | ε }**

d) **It is ambiguous as we can find more than one leftmost derivation**

**e.g.**

**S -> SaSaSbS -> aSaSbS -> aaSbS -> aabS -> aabSaSaSbS -> aabaab**

**S -> SaSaSbS -> SaSaSbSaSaSbS -> aabaab**

3.

a) **X = {0n1n2n| n >= 0}**

**Y = {0n1n2m| n, m >=0}**

**X is not context free, as we can prove it by pumping lemma.**

**Y is context free and is nonregular, as we can define it with a PDA or CFG (we can concatenate 0n1n to 2m) but not a DFA or NFA.**

**X is a subset of Y as we can arbitrarily choose m to be n.**

b)

**A = {0n1n | n >= 0}**

**B = {0n1n2n U 0n1n | n >= 0}**

**A is nonregular as proved in the textbook.**

**B is context free and is nonregular, as we can define it with a PDA or CFG (we can concatenate 0n1n to 2m) but not a DFA or NFA.**

**A is a subset of B because it is included in the union of 0n1n2n and 0n1n**